ECON0106: **Microeconomics**

Problem Set 10

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Question 1. Consider the following stage game:

		Player 2			
		a	b	c	d
Player 1	A	$5,1$	$5,2$	$3,3$	$1,2$
	B	$6,10$	$9,9$	$4,11$	$3,9$
	C	$1,3$	$11,4$	$5,5$	$2,4$
	D	$4,4$	$8,3$	$3,5$	$1,10$

(i) For each playear, which strategies are rationalizable?

When treating repeated versions of this stage game below, assume the usual discounted payoffs with a common discount factor δ for both players.

(ii) Suppose the stage game is repeated a fininte number of times $T < \infty$, and $\delta \in [0,1]$. Characterize, as a function of T and δ, the set of subgame perfect Nash equilibria.

For the remainder of the question, assume the stage game is repeated infinitely many times and $\delta < 1$.

(iii) For what values of δ (if any) can the action pair (B,b) be sustained in every period of a subgame perfect Nash equilibrium using Nash reversion strategies? Describe the equilibrium strategy profile.

(iv) For what values of δ (if any) can (B,b) be sustained in every period of a Nash equilibrium? Be sure to specify the strategy profile you use.

(v) Argue that for any δ, there is no Nash equiibrium where the average discounted payoff profile is $(10,10)$.

Question 2. Consider a stage game in which there are n firms competing in prices. For a given price vector p, let $\underline{p} := \min_j p_j$. Demand for firm i is 0 if its price $p_i > \underline{p}$ is not the lowest, and it is given by $x(\underline{p})/|\arg\min_{j\in\{1,\dots,n\}} p_j|$ if $p_i = \underline{p}$, where x is a 'nice' function (say

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differentiable, decreasing, and logconcave — i.e. $\ln x(p_i)$ is concave). Each firm has a constant marginal cost of $c > 0$. Assume that $\exists \overline{p} \in \mathbb{R}_+ \cup \{\infty\}$ such that $\lim_{p \uparrow \overline{p}} |x'(p)|/x(p) = \infty$.

(i) Suppose there is only one period and one firm. Show that the monopoly price is well-defined.

Assume for the remainder of the question that the monopoly profit is strictly positive.

(ii) Suppose the game is repeated $T < \infty$ times and payoffs discounted according to a common discount factor $\delta \in [0, 1]$. Characterize the set of subgame perfect Nash equilibria. *(Hint: Consider first characterising the set of SPNE outcomes.)*

For the remainder of the question, assume the game is repeated infinitely many times and $\delta \in (0, 1)$.

(iii) What is the highest (average discounted) joint profit the industry (taking all firms together) can achieve? Use Nash reversion strategies and find the smallest discount factor that can support this profit level as a subgame perfect Nash equilibrium outcome.

(iv) Now suppose that market demand in period t demand is given by $x_t(p) := \gamma^t x(p)$. Using Nash reversion strategies, find the smallest discount factor that supports the monopoly price being observed in each period (as an outcome of a SPNE).

(v) Similarly, suppose that, at the end of each period, the market ceases to exist with probability $\alpha \in (0, 1)$. Using Nash reversion strategies, find the smallest discount factor that can sustain the monopoly price prevailing in each period (as an outcome of a SPNE).

(vi) Finally, suppose that it takes M periods to respond to a deviation. Again using Nash reversion strategies, find the smallest discount factor that can sustain monopoly price as an equilibrium (as an outcome of a SPNE).

Question 3. Exercise 19 in Navin Kartik's notes (p. 67).